The Contraceptive Revolution and the Second Demographic Transition: 
An Economic Model of Sex, Fertility, and Marriage

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We present a model of household decisions regarding sex, fertility, marriage, and consumption. Households choose marital status based on the expected utility of marriage, and then sex, children, and consumption of other goods to maximize utility subject to a budget and a fertility constraint. An increase in contraceptive efficacy generally leads to increased sexual activity but has ambiguous effects on fertility. Also, increases in contraceptive efficacy lead to lower marriage rates and higher divorce rates. The predictions correspond to the features of the Second Demographic Transition, including declining overall fertility rates, increasing non-marital fertility, and the decline in marriage.

Keywords: microeconomic theory, contraception, sex, marriage, divorce, cohabitation, fertility

JEL Classification: D01, J12, J13, Z13

Introduction

The substantial decline in both mortality and fertility that began around 1960—referred to as the Second Demographic Transition\(^1\) by sociologists and demographers—has been widely acknowledged as costly to children’s welfare.\(^2\) Key features of this transition include rising marital instability (through cohabitation and divorce), enormous upward swings in non-marital childbearing, and delays in marriage and marital childbearing. These contrast sharply with the features of the First Demographic Transition—beginning in the early 1800s—that benefitted children through better health and physical welfare, greater familial stability, and increased access to education.

There are two economic puzzles related to the Second Demographic Transition. First, the decline in fertility has occurred during a long period of substantial economic growth, even though fertility should increase with the rising incomes, since children are thought to be a normal

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\(^1\) The term “second demographic transition” was originally coined by Lesthaeghe and Van de Kaa in a book chapter entitled “Twee demografische transities?” (Two demographic transitions?).

\(^2\) McLanahan, 2004
good. These contradictory trends have led some economists to question whether children are a normal good. Second, with the decrease in overall fertility there has been simultaneously an increase in non-marital childbearing. This increase comes despite the widespread availability of reproductive technologies, such as the Pill and legal abortion, to unmarried women since the late 1960s. In fact, non-marital childbearing has continued to increase despite the increasing effectiveness of these technologies over the last 50 years.

Figure 1. Percent of births to unmarried women, 1940-2006

Figure 1 illustrates the recent trends in non-marital child-bearing. From 1960 to 2006, the percent of all births to unmarried women in the United States increased 62.6%. Although the total percentage of unmarried births in the black community is twice that among white women, the actual percent increases have been largest among the non-black populations, indicating that

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3 While the FDA approved the first birth control pill (Enovid) in 1960, various state-level statutes banned the sale of the Pill even to married women until 1965 the U.S. Supreme Court overturned a Connecticut law prohibiting the sale of contraceptives. In 1972 the U.S. Supreme Court overturned a Massachusetts law prohibiting the sale of contraceptives to unmarried women. It is generally maintained that a majority of U.S. women had access to the Pill by the early 70s. See Goldin and Katz, 2002, and Bailey, 2009, for thorough treatment of the diffusion and adoption of the Pill among unmarried and married women respectively.

4 Brady, Martin, & Ventura, 2007; also, Ventura & Bachrach, 2001

5 Ibid.
the growth in non-marital fertility has not been driven by a minority population. Rather, whatever is driving the upward trend in non-marital births appears to affect the United States broadly and with respect to every racial and ethnic subgroup.

The trend in non-marital childbearing presents a serious socioeconomic problem. Children born to unmarried parents are more likely to be born into poverty, less likely to ever live with their two biological parents, and more likely to engage in various risky behaviors at an early age. They are also emotionally and educationally disadvantaged. This has been a concern for policy makers since as early as 1935 with the initiation of Aid to Dependent Children (ADC). Policy has been largely ineffective at reversing the broad trend in non-marital births. According to the US Census Bureau, approximately 17 million children are living with mothers only, nearly 25% of all children.

While there have been ongoing programs to reduce teen pregnancy for several decades at both state and local levels, most prominently through schools, non-marital childbearing is no longer primarily a teenage phenomenon. In 1970, half of non-marital births were to women under the age of 20; today only 23% of non-marital births are to teenagers, and the biggest increases in non-marital fertility since 1970 have been in adult populations. Unfortunately, children born to older unmarried women do not fare better than those born to unmarried teenage women and the socioeconomic consequences are not substantively different.

In the economic and sociological literature there are three approaches that attempt to reconcile the apparently contradictory trends of increasing non-marital childbearing and declining overall fertility. The first approach stipulates a common cause of the two trends, such

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6 Amato and Keith, 1991; McLanahan and Sandefur, 1994; Edin and Kefalas, 2005
8 Ventura, 2009
9 Geronimus and Korenman, 1992, 1993
as a cultural change, women’s wages, or the structure of welfare payments. Classic papers include Thornton, Axinn and Hill (1992), Lesthaeeghe (1983), Lesthaeeghe and Surkyn (1988), Lesthaeeghe and Wilson (1985), and Bumpass (1990). In the economics literature, Greenwood and Gruner (2010) have formalized the cultural change argument most recently. The second approach models the changes in birth control access as a technology shock that has different effects on different populations, generating in some instances the expected result (fewer births) and in other instances the counter-intuitive result (more births). The perennial paper is Akerlof, Yellen and Katz (1996), but sociologists have produced distinct and complementary work looking at differential unwanted fertility rates by educational status. A third approach argues that many non-marital births are not “unwanted” births, but may be better understood as “re-timed” births. Sociologists Edin and Kefalas have explored this possibility qualitatively, arguing that observed patterns of non-marital childbearing for low-income women should not necessarily be viewed as unwanted or ill-timed, but rather optimally timed given various life constraints. In their view, non-marital fertility is explained as a rational response to changing incentives. Geronimus (1997) and Geronimus and Korenman (1992) also support a re-timing hypothesis. However, none of the explanations of the second puzzle—the contradictory trends in fertility—explain the decline in overall fertility in the presence of economic expansion and rising incomes—the first puzzle. The accepted explanation for this is Becker (1960; 1981), who reconciles the income and fertility trends by arguing that, as incomes rise, households substitute quality for quantity of children. We propose an alternate model that solves both puzzles.

Beginning with a simple neoclassical model, we introduce a fertility constraint that relates sexual activity and births in household decisions regarding marriage, sex, children, and

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11 See especially Edin and Kefalas, 2005; also Edin 2000
consumption of other goods. Contraceptive technology affects the model by weakening the fertility constraint, so that households with more effective contraception can choose higher levels of sexual activity for a given number of children, or fewer children for a given amount of sexual activity. We model advancements in contraceptive technology, such as the Pill, or access to abortion, as exogenous changes in the efficacy of contraception. We demonstrate that the effect of an increase in contraceptive effectiveness on household fertility will depend on ex ante household sexual activity and that a modest elasticity of fertility with respect to contraceptive effectiveness reconciles the trends of rising incomes and declining fertility while maintaining the assumption that children are normal goods. Furthermore, we extend the neoclassical model to incorporate the decision to marry and demonstrate that an increase in contraceptive effectiveness can explain the other characteristics of the Second Demographic Transition—later ages of first marriage, lower overall marriage rates and higher divorce rates, or what is referred to as the “ Retreat from Marriage” in sociological literature.

Model

The model is separable into two stages: the second-stage model is a model for demand for sex, children, and other goods, and the first-stage model is a model of the choice to marry or not. The decision to marry is determined by the difference in utilities between married and unmarried households. The difference in utilities is ordinarily generated by the income effect, as married households have more resources, or income, than unmarried households, which leads to higher utility and higher demand for sex, children, and other goods, all of which are assumed to be normal.
Sex and Children

Let $s$, $k$, and $x$ be demand for sex, children (kids), and other goods, respectively, with corresponding prices $p_s$, $p_k$, and $p_x$. Let $m$ be income, $\beta$ be the natural productivity of sex, and $z$ be the efficacy of contraception, $z \in [0,1]$, with $z = 0$ being no contraception and 1 being sterilization. An increase in $z$ means an increase in the efficacy of contraception. Consider the household’s problem

$$v(p, m, z) = \max_{s,k,x} u(s, k, x)$$

subject to

$$\begin{align*}
& p_s s + p_k k + p_x x \leq m \\
& (1 - z) \beta s \leq k.
\end{align*}$$

Households choose sex, children, and consumption of other goods so as to maximize utility subject to the budget constraint and a fertility constraint relating sex to the number of children they desire to have. Non-negativity constraints on sex, children, and other goods are implicit. The indirect utility function is a function of prices, income, and contraceptive efficacy, which is exogenous to the model. We assume that a solution exists for all choices of $z$. With complete sterilization, $z = 1$ and the problem reduces to the ordinary consumer’s problem.

What is meant by contraceptive efficacy, $z$? This parameter includes not just the technological efficacy, but also knowledge of how to use contraceptive technology, the availability of such technology to households, and the willingness to use it. The efficacy of contraception increases as technology improves, as it becomes more widely available, and as households are more willing and able to use it. A corollary will be that the contraceptive efficacy parameter also increases with the multiplication of family planning clinics and with the proliferation of sex education classes when such education includes components to teach

\[12\] Knowledge and education are an important part of this parameter.
students how to use contraception, informs them where it is available, and encourages them to use it.\footnote{There is a strong correlation between education and willingness to use contraception—this is robust across countries.}

Let $\lambda$ and $\mu$ be the Lagrange multipliers on the budget and the fertility constraint, so the Lagrangian is

$$L = u(s, k, x) + \lambda(m - p_s s - p_k k - p_x x) + \mu(k - (1 - z) \beta s).$$

In addition to the constraints, the first-order conditions for this problem are

$$u_s - \lambda p_s - \mu (1 - z) \beta = 0$$
$$u_x - \lambda p_x = 0$$
$$u_k - \lambda p_k + \mu = 0.$$

The effective price of sex includes the price $p_s$ plus the implicit cost of children, and the effective price of children is the direct cost $p_k$ minus the indirect benefits of sex. However, the direct costs of sex, $p_s$, are likely to be trivially small, so decisions about sex will tend to be dominated not by the price of sex but the implicit cost of children. Notice that the fertility constraint effectively increases the effective price of sex and decreases the effective price of children. This leads to the following ordering

$$\frac{u_s}{p_s} > \frac{u_x}{p_x} > \frac{u_k}{p_k}.$$

When $z < 1$ and the fertility constraint is binding, this ordering of marginal utilities per dollar must hold for all feasible prices in order for a maximum to exist. In particular, this order must hold in the state of nature, i.e. when no contraception is used, or $z = 0$. If this ordering is a property of utility functions in general, then the implications of sterilization for the model are readily apparent: sterilization removes the fertility constraint, so utility maximizing households will have as much sex as possible, spend the remainder of the budget on consumption of other
goods, and have no children. This is true of both sterilization and of any contraceptive practice that is as efficacious as sterilization.

How does demand for sex change as contraception becomes more effective? This change can represent either the introduction of a new contraceptive technology, such as the Pill, or the broad historical trend of increasing contraceptive availability, education, and efficacy observed especially over the past two centuries. Let $e^H(p, v, z)$ be expenditure function for the corresponding expenditure minimization problem with associated solution variables $(s^H, k^H, \xi^H, \theta^H, \eta^H)$. The effect of contraceptive effectiveness on the demand for sex is

$$\frac{\partial s^*}{\partial z} = \frac{\partial s^H}{\partial z} + \beta \eta^H \frac{\partial s^*}{\partial m}$$

This expression decomposes the effect of contraception on demand for sex into what we call a contraceptive substitution effect and an income effect. Both terms are theoretically ambiguous, but there are good reasons to believe they are both positive. The contraceptive substitution effect can be decomposed so the expression becomes

$$\frac{\partial s^*}{\partial z} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_s} - \beta \eta^H \frac{\partial s^H}{\partial p_s} + \beta \eta^H s^* \frac{\partial s^*}{\partial m}$$

The three terms are the implicit cost effect, the substitution effect, and the income effect, respectively. The implicit cost effect is how the implicit cost of children—the Lagrange multiplier of the fertility constraint—changes as the price of sex increases. This term is theoretically ambiguous but is expected to be negative empirically, i.e. $\frac{\partial \eta^H}{\partial p_s} > 0$. The second term is the ordinary substitution effect; $\frac{\partial s^H}{\partial p_s} \leq 0$ so the second term is nonnegative. Furthermore, we expect the substitution effect to dominate the implicit cost effect, so $\frac{\partial s^*}{\partial z} \geq 0$ in the first equation.
The income effect is positive if sex is a normal good. We share the consensus that inferior goods are members of broader categories of goods that are normal in aggregate. Since sex is not a member of either of the other two categories, it is a normal good.\(^{14}\)

Do marginal changes in contraceptive efficacy lead to a relatively greater impact on demand for sex in unmarried households or married ones? The first-order conditions to the utility maximization can be rearranged to show

\[
\lambda = \frac{u_x}{p_x}
\]

\[
\mu = \frac{u_x}{p_x} p_k - u_k
\]

Substituting these into the first-order condition for sex,

\[
u_s - \lambda p_s + (1 - z)\beta [u_k - \lambda p_k] = 0
\]

From the earlier analysis, \(u_s - \lambda p_s > 0\) and \(u_k - \lambda p_k < 0\): sex is a net benefit and children are a net cost. Households for whom children are a net benefit do not use contraception. From this equation, the first-order effect of an increase in contraceptive efficacy is to reduce the net cost of children. Changes in contraceptive efficacy will have a greater effect on households for which the net cost of children is higher, and a lesser effect on those for which the net cost of children is lower. Generally, the net costs of children will be higher for unmarried households and lower for married ones, so marginal improvements in contraception should lead to a larger change in demand for sex among unmarried households than it will among married ones. If household consumption of other goods exhibits diminishing marginal utility, for example, then the marginal utility of consumption of other goods, \(\lambda\), will be lower for married households because of their higher income. Married households would then have lower direct costs of sex and indirect costs of children, so they will tend to have more sex and children than unmarried households.

\(^{14}\) Since sex is not an inferior member of children or other goods, so it is normal. This is unlike, say, fast food, which is an inferior member of restaurant meals, which in aggregate is a normal good.
We now turn to how demand for children is affected by an increase in the efficacy of contraception. Like demand for sex, the changes in demand for children in response to increasing the efficacy of contraception can be separated into a contraceptive substitution effect and an income effect,

\[ \frac{\partial k^*}{\partial z} = \frac{\partial k^H}{\partial z} + \beta \eta^H s^* \frac{\partial k^*}{\partial m} \]

Both terms are theoretically ambiguous but the income effect is positive if children are a normal good. This should be true for the same reasons given earlier, i.e. children are not an inferior member of the categories of sex or other goods, so they should be normal. As with the demand for sex, the demand for children can be decomposed into an implicit cost effect, a substitution effect, and an income effect, so this expression becomes

\[ \frac{\partial k^*}{\partial z} = -\beta s^* \frac{\partial \eta^H}{\partial p_k} - \beta \eta^H \frac{\partial s^H}{\partial p_k} + \beta \eta^H s^* \frac{\partial k^*}{\partial m} \]

The implicit cost effect and the substitution effect are both theoretically ambiguous.

The fertility constraint can clarify the ambiguity in the change in demand for children in response to an increase in contraceptive efficacy. Assuming that the fertility constraint is binding before and after any changes, optimal demand for children is given by

\[ k^* = (1 - z) \beta s^* \]

Taking the derivative with respect to the efficacy of contraception, \( z \), shows

\[ \frac{\partial k^*}{\partial z} = -\beta s^* + (1 - z) \beta \frac{\partial s^*}{\partial z} \]
The first term is negative, and the second term is positive;\(^{15}\) the result is theoretically ambiguous, but depends on the amount of sex prior to the change, \(s^*\). In particular,

\[
\frac{\partial k^*}{\partial z} < 0 \text{ when } s^* > (1 - z) \frac{\partial s^*}{\partial z} \text{ and }
\]

\[
\frac{\partial k^*}{\partial z} > 0 \text{ when } s^* < (1 - z) \frac{\partial s^*}{\partial z}.
\]

The effect of a change in contraceptive efficacy depends on sexual activity of the household prior to the change: fewer children will be born to households that were more sexually active before the change, and more children will be born to households that were less sexually active before the change. This analysis is valid when demand for sex depends on contraceptive efficacy and the fertility constraint is binding, but it is otherwise independent of the household’s utility maximization problem, so the result is very general. The Hicksian demands for sex and children are related in the same manner, for example.

For the special case of \(s^* = 0\), i.e. households without children, an increase in contraceptive effectiveness unambiguously leads to an increase in children. Clearly, \(\frac{\partial k^*}{\partial z} > 0\) for \(s^* = 0\), i.e. increasing the efficacy of contraception will lead to more children being born to households that would have otherwise been abstinent. This case may describe unmarried households when contraception is either unavailable or ineffective, e.g. society prior to the Pill. The model predicts an increase in children for these households and a decrease in children for households with a high \(s^*\), e.g. married households. Associating low \(s^*\) with unmarried households and high \(s^*\) with married households, these predictions of the model conform to the general observation that

\(^{15}\) The second term is theoretically ambiguous but likely positive for the reasons given earlier.
out-of-wedlock childbirths have increased since the invention of the Pill while at the same time
the overall fertility rate has decreased.\(^{16}\)

Generally, since demand for sex increases with income and decreases with the cost of
children, married households would have had more sex than unmarried households prior to the
change in the efficacy of contraception. The two results, then, may be assumed to correspond to
the married and unmarried households: for married households, \(s^*\) is relatively high and \(\frac{\partial k^*}{\partial z} < 0\);
for unmarried households, \(s^*\) is relatively low and \(\frac{\partial k^*}{\partial z} > 0\). So the model predicts that
technology shocks which increase the efficacy of contraception should lead to fewer children
born to married households and more children born to unmarried households. Where married
households are a much larger fraction of the population, the net effect of these two changes
would be a decline in the overall fertility rate. This is consistent with recent historical trends in
both fertility and out-of-wedlock births.

The comparative statics results of the effect of contraception on the demand for sex and
children can be expressed in terms of budget shares and elasticities. Let \(\varepsilon_{s,z}^*\) be the elasticity of
demand for sex with respect to contraceptive ineffectiveness,

\[
\varepsilon_{s,z}^* = \frac{\partial s^* (1 - z)}{\partial z} \frac{s^*}{s^*}
\]

Let \(\varepsilon_{s,m}^H\) be the analogous elasticity of Hicksian demand for sex with respect to contraceptive
ineffectiveness, \(\varepsilon_{s,m}^*\) be the income elasticity of sex, and \(\omega_k^H\) be the implicit budget share of
children, \(\omega_k^H = \frac{\eta^H k^*}{m}\). The elasticity of demand for sex with respect to contraceptive effectiveness
is

\(^{16}\) The overall fertility rate would be dominated by the fertility changes to married households, which outnumber
unmarried ones.
The income elasticity is positive and the Hicksian elasticity, while theoretically ambiguous, should be positive empirically, so the elasticity of demand for sex with respect to contraceptive effectiveness should be positive empirically. Let $\varepsilon_{s,x}^*$ be the elasticity of demand for children with respect to contraceptive ineffectiveness, $\varepsilon_{k,x}^H$ be the analogous elasticity of Hicksian demand, and $\varepsilon_{k,m}^*$ be the income elasticity of demand for children. The elasticity of demand for children with respect to contraceptive effectiveness is

$$\varepsilon_{k,x}^* \equiv \varepsilon_{k,x}^H + \omega_k^H \varepsilon_{k,m}^*$$

Here the income elasticity is positive but the Hicksian elasticity of demand for children is theoretically ambiguous, so the elasticity of demand for children with respect to contraceptive effectiveness is ambiguous.

**Marriage**

We now extend the model to include the decision to marry. Why do households, or couples, choose to marry? The argument here is that households marry because of the income effect. Income includes the value of time spent either in the labor market or in the production of household goods, and the demands of children are such that the additional income or resources of another parent are desired to assist a household in satisfying those demands. Unmarried households are more likely to have fewer children or to not have children because they cannot afford them, and married households will tend to have more children because they can afford them.\(^{17}\)

\(^{17}\) There are some critical exceptions to this, discussed below.
Households face the same prices—$p_s$, $p_r$, and $p_x$—and the same contraceptive efficacy, $z$.

Let $m_1$ be the income of unmarried households and $m_2$ the income of married households, $m_2 > m_1$. Since the indirect utility function is increasing in income, the utility of married households will be higher than that of single households, and, in the deterministic model, all households would marry. Let $\varepsilon_1$ and $\varepsilon_2$ be random variables that describe the unpredictable nature of unmarried and married life, respectively, and let $\varepsilon = \varepsilon_2 - \varepsilon_1$. The values of the random variables are known prior to the marriage decision. Assume that $E(\varepsilon_1) = E(\varepsilon_2) = 0$, so $E(\varepsilon) = E(\varepsilon_2 - \varepsilon_1) = 0$. Let $w$ be an indicator variable of marital status where $w = 1$ indicates marriage, so households marry according to

$$\max_w (1 - w)[v(p, m_1, z) + \varepsilon_1] + w[v(p, m_2, z) + \varepsilon_2]$$

Households marry when

$$v(p, m_1, z) + \varepsilon_1 < v(p, m_2, z) + \varepsilon_2$$

or

$$v(p, m_1, z) - v(p, m_2, z) < \varepsilon$$

We will assume that the derivative of the indirect utility function with respect to income is continuous, so, for some $m_1 \leq m' \leq m_2$ we can express the marriage condition as

$$\frac{\partial v(p, m', z)}{\partial m} (m_1 - m_2) < \varepsilon$$

Households will not marry when

$$\frac{\partial v(p, m', z)}{\partial m} (m_1 - m_2) > \varepsilon$$

Let $y(z)$ be the difference in utility between single and married households for a given level of contraceptive efficacy, $y(z) = v(p, m_1, z) - v(p, m_2, z)$, and let $F(t)$ be the cumulative distribution function of $\varepsilon$, where $F'(t) > 0$. Because married households have higher income,
they have higher utility on average, \( y(z) < 0 \) and \(-y(z)\) is the average utility premium that married households receive over unmarried ones. The fraction of unmarried households is the integral of \( \varepsilon \) from negative infinity to the point where \( t = y(z) \) and the integral from that point to positive infinity is the fraction of married households, i.e. \( pr(\text{unmarried}) = pr(w = 0) = F(y(z)) \) and \( pr(w = 1) = 1 - pr(w = 0) \). This static model can be thought of as a snapshot of society over a particular time period: the first inequality describes households that marry or remain married; the second households that remain unmarried or divorce.

Figure 2 shows the effect of increasing contraceptive efficacy on marriage rates. The points \( y(z) \) and \( y(z') \) show the differences in unmarried and married household utility levels for different levels of contraceptive efficacy, \( z' > z \). As contraception becomes more effective, the utility premium of marriage is reduced, so there is less reason to get and stay married and more reason to remain or become unmarried. This corresponds to a number of trends in marriage: later ages of first marriages, lower marriage rates, and higher divorce rates, i.e. the Retreat from Marriage.
The decision to marry or not depends on the difference between the utility of married and unmarried households. An increase in contraceptive effectiveness leads to fewer married households and more unmarried households—single or divorced—when the marginal utility of the efficacy of contraception is greater for unmarried households than it is for married households. From earlier, \( pr(\text{unmarried})= F(y(z)) \), so an increase in contraceptive effectiveness increases the probability of a household being unmarried when

\[
F'(s) > 0, \text{ so the inequality is satisfied when the bracketed term is negative. Applying the Envelope Theorem results to this equation, an increase in the efficacy of contraception will lead to fewer married households and more unmarried households—single or divorced—when}
\]

\[
\beta \left[ \frac{\partial \mu^*}{\partial m} s^*(p, m_1, z) + \frac{\partial s^*}{\partial m} \mu^*(p, m_1, z) \right] < 0.
\]

Let \( \varepsilon_{s,m}^* \) be the income elasticity of sex and \( \varepsilon_{\mu,m}^* \) be the income elasticity of the implicit cost of children, i.e. the Lagrange multiplier. An increase in contraceptive effectiveness leads to fewer married households and more unmarried households when

\[
\varepsilon_{\mu,m}^* + \varepsilon_{s,m}^* < 0
\]

Sex is a normal good, so its income elasticity is positive. This inequality is satisfied if the income elasticity of the implicit cost of children is negative and larger in magnitude than the income elasticity of sex.

The effect of contraception on marriage rates depends critically on the income elasticity of the implicit cost of children, \( \varepsilon_{\mu,m}^* \). While the sign of this elasticity is theoretically ambiguous, there are a number of reasons to believe this elasticity is empirically negative and substantial. First, if income exhibits diminishing marginal utility among household members, then a child
would be less costly in terms of the disutility of forgone consumption at higher income levels. This would cause the implicit cost of children to fall with income. Additionally, the only difference between married and unmarried households in the model is income, so the income elasticity incorporates the effects of both income and marital status on the implicit price of children. The implicit cost of children could be lower among married households—with higher pooled income—because they believe that a family is the right environment within which to raise a child, that it is easier to raise a child if one is married, and that children are happier and better off if both parents are present in the household. It is also possible that marital status affects household decisions independently of income, and that the utility functions of married and single households are fundamentally different. A theoretical model incorporating the effects of marital status independent of income would be an important extension of this research.

Cohabiting households (if they share income) are classified here as married in this model because we model the decision to marry as the decision to pool income. Of course, formally married couples are distinguished from cohabiting couples by a public promise of lifelong fidelity, but in other respects households of cohabiting couples are indistinguishable from those that are formally married. Empirically, cohabiting households are less stable than formally married households (Brown & Booth, 1996; Bumpass & Sweet, 1989). If cohabitation can be thought of as a less stable form of marriage, then the prediction that increasing contraceptive efficacy leads to increased marital instability means that it will lead to fewer formally married households and more cohabiting ones.

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If marital status affected households independently of income, married households would be expected to have higher fertility than unmarried households when faced with the same prices, income, and levels of contraceptive effectiveness.
Comparing the Income and Contraception Effects

We use the Marshallian demand for children to estimate the elasticity of demand for children with respect to contraceptive effectiveness, or the contraception elasticity of fertility. The homogeneity of the Marshallian demand \( k^*(p, m, z) \equiv k^*(\bar{p}, \bar{m}, z) \), where \( \bar{m} = m / \bar{p} \) and \( \bar{p} = p / \hat{p} \) for some price index \( \hat{p} \). The total derivative of the Marshallian demand is

\[
dk^* = \frac{\partial k^*}{\partial \bar{p}} d\bar{p} + \frac{\partial k^*}{\partial \bar{m}} d\bar{m} + \frac{\partial k^*}{\partial z} dz
\]

For the moment we will hold the real price constant and focus on the income and contraception effects. The derivative can be expressed in percentage terms as

\[
\frac{dk^*}{k^*} = \frac{\partial k^*}{\partial \bar{m}} \bar{m} \frac{d\bar{m}}{\bar{m}} + \frac{\partial k^*}{\partial z} (1 - z) \frac{dz}{(1 - z)}
\]

or

\[
\frac{dk^*}{k^*} = \frac{\partial k^*}{\partial \bar{m}} \bar{m} + \frac{\partial k^*}{\partial z} (1 - z)
\]

We use this equation for our estimates.

**Table 1. Fertility, Income, and Contraceptive Effectiveness, 1965 and 2000**

<table>
<thead>
<tr>
<th>Total Fertility Rate (TFR)</th>
<th>Crude Birth Rate</th>
<th>Income per Family</th>
<th>Annual Failure Rate (AFR)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>% ch</td>
<td>-41.9%</td>
<td>-38.8%</td>
<td>62.6%</td>
</tr>
<tr>
<td>2000</td>
<td>2.05</td>
<td>13.9</td>
<td>78,867</td>
</tr>
<tr>
<td>1965</td>
<td>3.53</td>
<td>22.7</td>
<td>48,509</td>
</tr>
</tbody>
</table>

* % change for AFR is calculated as \( dz / (1 - z) \)

Sources: Haines, Greenwood and Gruner, Census

Table 1 summarizes the data used in these calculations. Fertility is measured using the total fertility rate (TFR), which is the expected children per woman over the course of her life given
contemporary birth rates. The total fertility rate was 3.53 in 1965 and 2.05 in 2000, a decrease of 41.9%. An alternative measure of fertility is crude birth rates, i.e. births per 1000 people per year. The crude birth rate decreased by 38.8% from 1965 to 2000, from 22.7 to 2000. Data on real income per family measured in 2009 dollars is from the U.S. Census Bureau. Real income was $48,509 per family in 1965 and $78,867 in 2000, an increase in real income of 65.6%.

Contraceptive effectiveness is measured by the annual failure rate for prevailing contraceptive methods. Our estimates are taken from Greenwood and Gruner (2010). The annual failure rate decreased from 17.5% in 1965 to 5.5% in 2000, a 68.8% decrease. The endpoints were chosen because the Pill was just beginning to be widely accepted and adopted in 1965 and the economy was growing but unaffected by the housing bubble and subsequent crash in 2000.

We use an income elasticity of 0.15 from Lindo’s (2010) analysis of income shocks. Other available income elasticities of children are negative, but this can be attributed the fact that these use macro-level data and omit contraception effects. Lindo utilizes a quasi-natural experiment with micro-level data. His results are also consistent with economic theory and his short-term experimental design would not have been affected by bias from long-term changes to contraceptive technology.

<table>
<thead>
<tr>
<th>Income Elasticity</th>
<th>Income per Family</th>
<th>Contraception Elasticity</th>
<th>Annual Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Fertility Rate</td>
<td>-41.9%</td>
<td>0.15</td>
<td>62.6%</td>
</tr>
<tr>
<td>Crude Birth Rate</td>
<td>-38.8%</td>
<td>0.15</td>
<td>62.6%</td>
</tr>
</tbody>
</table>

Estimates of the contraception elasticity of fertility are provided in Table 2. We estimate a contraception elasticity of fertility of -0.748 using the total fertility rate. This means that a 10%
increase in contraceptive effectiveness relative to the annual failure rate reduces fertility by 7.48%. The contraception elasticity of fertility using the crude birth rate is -0.702. These estimates are a good starting point, but a more accurate estimate would incorporate a measure of the changes in the cost of children. Summary measures of the relative price or cost of children are not available, and we leave the construction of such a measure and a more accurate estimate of the contraception elasticity of fertility as an extension to this research.

In holding relative prices constant we have assumed that the price of children relative to the price of other goods has remained unchanged from 1965 to 2000. This would be the case, for example, if all goods are available in competitive markets and technological change is neutral and not biased towards either children or other goods. If technological change has been biased towards children, then their price relative to other goods would have fallen. Assuming that the income elasticity is accurate, the contraception elasticity provided here is as an upper bound to the true value, i.e. the true value, which is negative, is larger in magnitude. If technological change was biased towards other goods, then the true value is smaller in magnitude. However, given the broad definition of these categories and the broad effects of technological change, we don’t see any clear reason to conclude that technological change has been biased in either direction, and this provides some confidence in our assumption of constant relative prices.

Discussion

The three predictions of the model are that increasing contraceptive efficacy leads to increased demand for sex among all households, decreased demand for children among married households and increased demand among unmarried ones, and a decrease in the proportion of households that are married. Translating these predictions from the static context of the model to
historical data implies an increase in overall sexual activity, a decrease in the fertility rate, an
increase in non-marital births, delayed age of first marriage, fewer formally married households
and increased marital instability—all of the trends which have been identified as part of the
Second Demographic Transition.

First, the model predicts that an increase in contraceptive efficacy will lead to an increase in
sexual activity. An indirect measure of the level of sexual activity is the incidence of sexually
transmitted infections (STIs). Incidence reports of incurable viral STIs show strong upward
trends consistent with a significant increase in the level of sexual activity in the United States.
According to the CDC, there were 28.5 reported cases of genital warts per 100,000 women in
1966 and 78.0 reported cases in 2000. Reported cases of herpes also increased from 9.7 (per
100,000 women) in 1966 to 63.4 in 2000. Data on Chlamydia, human papilloma virus (HPV),
and HIV also show dramatic upward trends, though for HPV and HIV data from 1960 is not
applicable. Direct measures of sexual activity (survey data) also revealed upward trends in
reported sexual activity, especially among the young and unmarried.¹⁹

¹⁹ The sharpest increases were documented in the first decade after the Pill was made available. Data from the mid-
late 1980s showed a leveling off and even some modest declines in reported sexual intercourse. See Hofferth, Kahn
and Baldwin (1987) for a treatment of non-marital sexual activity in the post-Pill era, and also Goldin and Katz
(2000).
Second, the model predicts a decrease in the overall fertility rate. Figure 3 shows that the total fertility rate (TFR) has fallen by a third, from about three to two children per woman, since the 1960s. There is a vigorous debate about whether the “normal” pre-Pill TFR would have been more like the wartime lows precipitated by the first and second world wars, or more like the post-war boom, or something different altogether. Without taking a position on the wartime patterns, we argue that the contraceptive revolution should be credited with the post-1960s downward slope in fertility. Bailey (2009) provides careful empirical support for this hypothesis utilizing state-level variation in access to the Pill.

Third, the model predicts an increase in non-marital births. As discussed in the introduction, the percent of live births to unmarried mothers increased from 10.7% in 1970 to 33.2% in 2000 and then to 36.9% in 2005. The non-marital birthrate has increased not just for the nation as a whole, but for every racial category of women. Among white women, the percent of children born to unmarried women increased from 5.5% in 1970 to 31.7% in 2005, a six-fold increase; for

20 Source: Haines, 2008.
black women, the percent nearly doubled from 37.5% to 69.3%; and for Hispanic women, the rate more than doubled, from 23.6% in 1980 to 48% in 2005. These trends are consistent with the predictions of the model, since all women have had access to increasingly effective contraception for the time period.

Finally, this model predicts later marriage rates, lower marriage rates, increased marital instability (more divorce and more cohabitation). These predictions are also consistent with the historical trends since 1950. Bramlett and Mosher (2002) state that

In the United States during second half of the twentieth century, the proportion of people’s lives spent in marriage declined due to postponement of marriage to later ages and higher rates of divorce. The increase in non-marital cohabiting has also contributed to the decline in the proportion of peoples’ lives spent in marriage. Increasing rates of cohabitation have largely offset decreasing rates of marriage.

This observation by Bramlett and Mosher not only confirms that the predictions of the model are consistent with historical trends, but they also situate those trends in the second half of the twentieth century, which is consistent with our hypothesis that these trends may be seen as the outcome of utility-maximizing households in the presence of a relaxed fertility constraint due to the contraceptive revolution of the 1960s.

Conclusion

The model developed here presents a description of how marginal changes in contraceptive efficacy might affect household decisions regarding marriage, sex, and children in a neo-classical framework. Increased contraceptive efficacy increases demand for sex across both married and unmarried households. This yields more children among unmarried households and fewer children among married ones, generating both an increase in non-marital births and a plausible

---

decrease in the overall fertility rate. Finally, a significant increase in contraceptive efficacy reduces the utility differential between married and unmarried couples, which is expressed in later ages of first marriage, lower marriage rates, higher divorce rates, and increased rates of cohabitation. Each of these predictions of the model is consistent with the characteristics of the Second Demographic Transition and the associated Retreat from Marriage. In spite of its shortcomings, the model presented here seems to provide a plausible explanation for the puzzling demographic trends.

There are several limitations to the current model that provide opportunity for further research. In the model, households have children because of the income effect. If the income effect was the only reason that households had children, then (1) wealthier single households should still have more children than poor single households, and (2) a wealthy single household will have as much demand for children as a poor married household for some combination of income levels. Neither one of these implications of the model seems to correspond to the facts, suggesting that, while the model captures something important, it is still missing something. One potential solution is to consider the interaction of marriage with children, which could perhaps be modeled by incorporating marital status into the utility function of the household directly.

Another possible extension of the model would be to introduce differential access to marriage by income level. Many sociologists argue that poor women often face extreme obstacles in finding suitable marriage partners in contrast with their wealthier peers.23 This could be done by correlating the expected variance of marital income shocks with the individual income levels: high earning individuals expect a fairly stable (low-variance) marital income, while low-earning individuals might expect a more variable marital income. This would correlate non-marriage with poverty and non-marital births to poor households.

23 Among others, see Edin and Kefalas, 2005.
Another limitation of the model is the treatment of contraceptive efficacy as an exogenous variable. Making this variable endogenous would be an important extension of this research. Were contraceptive efficacy endogenous in the current version of the model, a household would always choose the maximum available because doing so minimizes the indirect costs of the fertility constraint to household utility and it can be done without cost. However, many forms of contraception, while relatively inexpensive, have high indirect costs, such as diminishing the enjoyment of sex itself (e.g. condoms) or other undesirable side effects (e.g. IUDs). There may be trade-offs between contraceptive efficacy, the marginal utility of sex, and other side effects which determine which form of contraception households decide to use. This trade-off could be captured by incorporating contraceptive efficacy into the household utility function and the budget constraint. It also suggests the existence of an efficient “contraceptive frontier,” where forms of contraception away from the frontier become obsolete over time, e.g. intrauterine devices (IUDs).
Bibliography


Appendix.

Utility Maximization

Model

\[ v^*(p, m, z) = \max_{s,k,x} u(s, k, x) \]
\[ s.t. \quad p_s s + p_k k + p_x x \leq m \]
\[ (1 - z)\beta s \leq k \]

Let \( \lambda \) and \( \mu \) be the Lagrange multipliers on the budget and the fertility constraint.

Lagrangian

\[ L = u(s, k, x) + \lambda (m - p_s s - p_k k - p_x x) + \mu (k - (1 - z)\beta s) \]

First-Order Conditions

The first-order conditions are

\[ u_s - \lambda p_s - \mu (1 - z)\beta = 0 \]
\[ u_x - \lambda p_x = 0 \]
\[ u_k - \lambda p_k + \mu = 0. \]

plus constraints.

The effective prices of sex and children should both be positive, so

\[ \lambda p_s + \mu (1 - z)\beta \geq 0 \]
\[ \lambda p_k - \mu \geq 0 \]

However, \( \mu \geq 0 \), so the first inequality is always satisfied. This leads to the restriction on the implicit cost of children

\[ 0 \leq \mu \leq \lambda p_k \]

Solution

\( v^*(p, m, z) \) indirect utility function

\( s^*(p, m, z) \) (Marshallian) demand for sex
\[ k^*(p, m, z) \quad (\text{Marshallian demand for children}) \]

\[ x^*(p, m, z) \quad (\text{Marshallian demand for other goods}) \]

From the first-order conditions, \( \mu \) is added to the cost of sex, \( p_s \), and subtracted from the cost of children, \( p_k \). The Lagrange multiplier \( \mu \) is the implicit cost of children in terms of more children and the implicit benefit of children in terms of sex.

**Expenditure Minimization**

**Model**

\[
e^H(p, v, z) = \min_{s,k,x} p_s s + p_k k + p_x x
\]

\[
s.t. \quad v \leq u(s, k, x)
\]

\[
(1 - z)\beta s \leq k
\]

Let \( \theta \) and \( \eta \) be the Lagrange multipliers on the budget and the fertility constraint.

**Lagrangian**

\[
L = p_s s + p_k k + p_x x + \theta(v - u(s, k, x)) + \eta((1 - z)\beta s - k)
\]

**First-Order Conditions**

The first-order conditions are

\[
p_s - \theta u_s + \eta(1 - z)\beta = 0
\]

\[
p_x - \theta u_x = 0
\]

\[
p_k - \theta u_k - \eta = 0
\]

plus constraints.

The effective prices of sex and children should both be positive, so

\[
p_s + \eta(1 - z)\beta \geq 0
\]

\[
p_k - \eta \geq 0
\]

However, \( \eta \geq 0 \), so the first inequality is always satisfied. This leads to the restriction on the implicit cost of children

\[ 0 \leq \eta \leq p_k \]
Solution

\[ e^H(p,v,z) \quad \text{expenditure function} \]

\[ s^H(p,v,z) \quad \text{Hicksian demand for sex} \]

\[ k^H(p,v,z) \quad \text{Hicksian demand for children} \]

\[ x^H(p,v,z) \quad \text{Hicksian demand for other goods} \]

The Lagrange multiplier \( \eta \) is the implicit cost of children in terms of more children and the implicit benefit of children in terms of sex.

Properties of the Expenditure Function

The expenditure function \( e^H(p,v,z) \) is

\( a) \) homogenous of degree 1 in \( p \),

\( b) \) nondecreasing in \( p \) and \( v \) separately,

\( c) \) nonincreasing in \( z \),

\( d) \) concave with respect to \( p \).

Proof

Homogeneity is from the definition of the expenditure function. From the Envelope Theorem

\[ \frac{\partial e^H}{\partial p_s} \equiv s^H(p,v,z) \geq 0 \]

\[ \frac{\partial e^H}{\partial p_k} \equiv k^H(p,v,z) \geq 0 \]

\[ \frac{\partial e^H}{\partial p_x} \equiv x^H(p,v,z) \geq 0 \]

Applying Envelope Theorem on the expenditure function applied to \( z \) and \( p_s \) shows
\[ \frac{\partial e^H}{\partial z} = -\beta \eta^H s^H(p, v, z) \equiv -\beta \eta^H \frac{\partial e^H}{\partial p_s} \leq 0 \]

The effect of \( z \), contraceptive effectiveness, is the price effect multiplied by a (negative) constant. Contraceptive effectiveness affects the expenditure function the same way that the price of sex does, but in the opposite direction. Contraceptive ineffectiveness, \((1 - z)\), is analogous to the price of sex.

To prove the concavity of the expenditure function in \( v \), observe that, for all \( v \),

\[ e^H(p, v, z) \leq (p_s s + p_k k + p_x x) \]

The primal-dual problem is

\[ \max_p e^H(p, v, z) - (p_s s + p_k k + p_x x) \]

The primal-dual problem achieves its maximum of zero at solution to the expenditure problem. Since the objective function of the original expenditure function is linear in \( v \), \( e^H(p, v, z) \) must be concave in \( p \) and \[ e^H_{pp} \] is negative semidefinite. (Silberberg, p. 165.)

**Properties of the Hicksian Demands**

The Hicksian demands are

a) homogenous of degree zero in \( p \)

b) nonincreasing in their own prices: \( s^H \) in \( p_s \), \( k^H \) in \( p_k \), and \( x^H \) in \( p_x \)

c) symmetric in other prices

**Proof**

These results come from the fact that the expenditure function is homogenous of degree 1 and concave in prices. For the Hicksian demand for sex,
Results for the Hicksian demand for children and other goods follow in the same manner.

**Hicksian Demand for Sex and Contraception**

The effect of contraceptive effectiveness on the Hicksian demand for sex is

\[
\frac{\partial s^H}{\partial p_s} \equiv \frac{\partial^2 e^H}{\partial p_s^2} \leq 0
\]

\[
\frac{\partial s^H}{\partial p_k} \equiv \frac{\partial^2 e^H}{\partial p_s \partial p_k} \equiv \frac{\partial^2 e^H}{\partial p_k \partial p_s} \equiv \frac{\partial k^H}{\partial p_s}
\]

\[
\frac{\partial s^H}{\partial p_x} \equiv \frac{\partial^2 e^H}{\partial p_s \partial p_x} \equiv \frac{\partial^2 e^H}{\partial p_x \partial p_s} \equiv \frac{\partial x^H}{\partial p_s}
\]

The first term is ambiguous and the second term is positive. The first term is referred to as the implicit cost effect and the second is the substitution effect. Furthermore, if and only if

\[
\frac{\partial \eta^H}{\partial p_s} \leq 0 \text{ or } \varepsilon_{s(H)\eta(H)} \leq -1,
\]

then

\[
\frac{\partial s^H}{\partial z} \geq 0
\]
If the implicit cost is decreasing with respect to the price of sex or the derivative is positive when Hicksian demand for sex with respect to its implicit cost is elastic, then Hicksian demand for sex is increasing in contraceptive effectiveness.

Proof

From the properties of the expenditure function

\[
\frac{\partial e^H}{\partial p_s} \equiv s^H(p, v, z)
\]

and

\[
\frac{\partial e^H}{\partial z} \equiv -\beta \eta^H s^H(p, v, z)
\]

Differentiating the first identity with respect to \( z \)

\[
\frac{\partial s^H}{\partial z} = \frac{\partial^2 e^H}{\partial p_s \partial z}
\]

Differentiating the second identity with respect to \( p_s \)

\[
\frac{\partial^2 e^H}{\partial z \partial p_s} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_s} - \beta \eta^H \frac{\partial s^H}{\partial p_s}
\]

By symmetry of the Hessian of the expenditure function

\[
\frac{\partial^2 e^H}{\partial p_s \partial z} \equiv \frac{\partial^2 e^H}{\partial z \partial p_s}
\]

Evaluating the fertility constraint at the optimum shows

\[
(1 - z) \beta s^H \equiv k^H
\]

so

\[
\beta s^H \equiv \frac{k^H}{(1 - z)}
\]

Combining these identities gives the first results.
Hicksian demand for sex is nondecreasing in contraceptive effectiveness when

\[-\beta s^H \frac{\partial \eta^H}{\partial p_s} - \beta \eta^H s^H \frac{\partial H}{\partial p_s} \geq 0\]

Cancelling out the \(\beta\) terms and rearranging shows

\[-s^H \frac{\partial \eta^H}{\partial p_s} \geq \eta^H s^H \frac{\partial \eta^H}{\partial p_s}\]

Dividing both sides shows

\[-\frac{\partial \eta^H}{\partial p_s} s^H \leq \frac{\partial \eta^H}{\partial p_s} \eta^H \leq 1\]

The left-hand side is the inverse of the elasticity of demand for sex with respect to its implicit cost

\[-\frac{1}{\varepsilon_{s(H)\eta(H)}} \leq 1\]

This elasticity is the elasticity of Hicksian demand for sex with respect to its implicit cost, so the ordinary properties of Hicksian demands suggest that \(\varepsilon_{s(H)\eta(H)}\) is negative. However, since

\[\frac{\partial s^H}{\partial p_s} \leq 0,\] this is true only if \[\frac{\partial \eta^H}{\partial p_s} > 0.\] Assume that \[\frac{\partial \eta^H}{\partial p_s} > 0,\] so \(\varepsilon_{s(H)\eta(H)} < 0.\) Multiplying both sides by \(\varepsilon_{s(H)\eta(H)}\) shows the result for the second case. Alternatively, if \[\frac{\partial \eta^H}{\partial p_s} < 0,\] then the elasticity is positive and the left-hand side of this inequality is negative and therefore satisfied. This is the first case.
If $\varepsilon_s(H)\eta(H) < -1$, then an increase in contraceptive effectiveness increases the Hicksian demand for sex. Given the earlier observation that contraceptive ineffectiveness is the effective price of sex, an increase in contraceptive effectiveness is analogous to a decrease in the price of sex. Under this assumption the Hicksian demand for sex is then downward sloping in its effective price.

The next section shows that there are strong reasons to believe that $\varepsilon_s(H)\eta(H) < 0$ and therefore $\varepsilon_s(H)\eta(H) \leq 0$.

**The Implicit Cost of Children**

The implicit cost of children can be calculated from the expenditure function as

$$\eta^H \equiv -\frac{1}{\beta} \frac{\partial e^H / \partial p_s}{\partial e^H / \partial z}$$

**Proof**

From the Envelope Theorem

$$\frac{\partial e^H}{\partial p_s} \equiv s^H(p, v, z)$$

$$\frac{\partial e^H}{\partial z} \equiv -\beta \eta^H s^H(p, v, z)$$

Dividing the second by the first gives the result.
The Implicit Cost of Children and the Price of Sex

The effect of the price of sex on the implicit cost of children can be calculated as

$$\frac{\partial \eta^H}{\partial p_s} \equiv - \frac{1}{\beta s^H} \left[ \frac{\partial s^H}{\partial z} + \beta \eta^H \frac{\partial s^H}{\partial p_s} \right]$$

This can be alternatively expressed as

$$\frac{\partial \eta^H}{\partial p_s} \equiv - \frac{e^H}{p_s k_H} \left[ w_s e^H s_{s,(1-z)} + \omega_k e^H s_{s,p(s)} \right]$$

Proof

$$\frac{\partial \eta^H}{\partial p_s} \equiv - \frac{1}{\beta} \left[ (\partial e^H / \partial p_s)(\partial^2 e^H / \partial z \partial p_s) - (\partial e^H / \partial z)(\partial^2 e^H / \partial p_s^2) \right]$$

$$\equiv - \frac{1}{\beta} \left[ (s^H)(\partial s^H / \partial z) - (\partial e^H / \partial z)(\partial s^H / \partial p_s) \right]$$

$$\equiv - \frac{1}{\beta} \left[ (s^H)(\partial s^H / \partial z) - (-\beta \eta^H s^H)(\partial s^H / \partial p_s) \right]$$

$$\equiv - \frac{1}{\beta} \left[ (\partial s^H / \partial z) + \beta \eta^H (\partial s^H / \partial p_s) \right]$$

$$\equiv - \frac{1}{\beta s^H} \left[ \frac{\partial s^H}{\partial z} + \beta \eta^H \frac{\partial s^H}{\partial p_s} \right]$$

Multiplying both sides by \( (1-z)p_s s^H e^H \) shows

$$\equiv - \frac{e^H}{(1-z)\beta p_s s^H} \left[ p_s s^H \frac{\partial s^H}{\partial z} \frac{1-z}{s^H} + \eta^H (1-z) \beta s^H \frac{\partial s^H}{\partial p_s} \frac{1-z}{s^H} \right]$$
\[ \equiv - \frac{e^H}{p_s k^H} \left[ p_s s^H \frac{\partial s^H (1 - z)}{\partial z} + \eta^H k^H \frac{\partial s^H}{\partial p_s} p_s \right] \]

\[ \frac{\partial \eta^H}{\partial p_s} \equiv - \frac{e^H}{p_s k^H} \left[ w_s^H \epsilon_s^H (1 - z) + \omega_k^H \epsilon_s^H p(s) \right] \]

**Proposition**

The implicit cost of children increases with the price of sex,

\[ \frac{\partial \eta^H}{\partial p_s} \geq 0 \]

**Argument**

From before,

\[ \frac{\partial \eta^H}{\partial p_s} \equiv - \frac{e^H}{p_s k^H} \left[ w_s^H \epsilon_s^H (1 - z) + \omega_k^H \epsilon_s^H p(s) \right] \]

The sign of the bracketed term will be the negative of the sign of the derivative, and is in general ambiguous. However, since the explicit budget share of sex, \( w_s^H \), is relatively small and the implicit budget share of children, \( \omega_k^H \), is relatively large, the bracketed term will tend to be dominated by \( \epsilon_s^H p(s) \). Since \( \epsilon_s^H p(s) \leq 0 \), it is expected that \( \frac{\partial \eta^H}{\partial p_s} \geq 0 \).

**The Implicit Cost of Children and the Price of Children**

The effect of the price of children on the implicit cost of children can be calculated as
\[
\frac{\partial \eta^H}{\partial p_k} \equiv -\frac{1}{\beta} \left[ \frac{\partial k^H}{\partial z} + \beta \eta^H \frac{\partial k^H}{\partial p_s} \right]
\]

This can be alternatively expressed as
\[
\frac{\partial \eta^H}{\partial p_k} \equiv -\frac{e^H}{p_s s^H} \left[ w^H_s \epsilon^H_k (1-z) + \omega^H_k \epsilon^H_{k,p(s)} \right]
\]

Both bracketed terms are ambiguous.

**Proof**

\[
\frac{\partial \eta^H}{\partial p_p} \equiv -\frac{1}{\beta} \left[ (\partial e^H/\partial p_s)(\partial^2 e^H/\partial z \partial p_k) - (\partial e^H/\partial z)(\partial^2 e^H/\partial p_s \partial p_k) \right]
\]

\[
\equiv -\frac{1}{\beta} \left[ (s^H) (\partial k^H/\partial z) - (\partial e^H/\partial z)(\partial k^H/\partial p_s) \right]
\]

\[
\equiv -\frac{1}{\beta} \left[ (s^H) (\partial k^H/\partial z) - (-\beta \eta^H s^H)(\partial k^H/\partial p_s) \right]
\]

\[
\equiv -\frac{1}{\beta} \left[ (\partial k^H/\partial z) - \beta \eta^H (\partial k^H/\partial p_s) \right]
\]

\[
\equiv -\frac{1}{\beta s^H} \left[ \frac{\partial k^H}{\partial z} + \beta \eta^H \frac{\partial k^H}{\partial p_s} \right]
\]

Multiplying both sides by \( \frac{(1-z)p_s s^H k^H e^H}{(1-z)p_s s^H e^H} \) shows

\[
\equiv -\frac{k^H e^H}{(1-z)\beta p_s s^H} \left[ \frac{p_s s^H \partial k^H (1-z)}{e^H \partial z k^H} + \frac{\eta^H (1-z) \beta s^H \partial k^H p_s}{e^H \partial p_s k^H} \right]
\]

\[
\equiv -\frac{e^H}{p_s s^H} \left[ \frac{p_s s^H \partial k^H (1-z)}{e^H \partial z k^H} + \frac{\eta^H k^H \partial k^H p_s}{e^H \partial p_s k^H} \right]
\]
\[
\frac{\partial \eta^H}{\partial p_s} \equiv - \frac{e^H}{p_{sH}} [w^H e^H_{k,(1-z)} + \omega^H_k e^H_{k,p(s)}]
\]

**Elasticity of Hicksian Demand for Sex and Contraception**

\[
\frac{\partial s^H}{\partial z} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_s} - \beta \eta^H \frac{\partial s^H}{\partial p_s}
\]

Multiply both sides by \(\frac{(1-z)p_s}{s^H \eta^H}\) yields

\[
\frac{\partial s^H}{\partial z} \frac{(1-z)p_s}{s^H \eta^H} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_s} \frac{(1-z)p_s}{s^H \eta^H} - \beta \eta^H \frac{\partial s^H}{\partial p_s} \frac{(1-z)p_s}{s^H \eta^H}
\]

\[
\varepsilon^H_{s,z} \frac{p_s}{\eta^H} \equiv - \frac{(1-z)\beta s^H}{s^H \eta^H} \varepsilon^H_{\eta,p(s)} - \frac{(1-z)\beta s^H}{s^H \eta^H} \varepsilon^H_{s,p(s)}
\]

\[
\varepsilon^H_{s,z} \frac{p_s}{\eta^H} \equiv - \frac{k^H}{s^H \eta^H} \varepsilon^H_{\eta,p(s)} - \frac{k^H}{s^H \eta^H} \varepsilon^H_{s,p(s)}
\]

\[
p_{sS^H} \varepsilon^H_{s,z} \equiv -\eta^H k^H \varepsilon^H_{\eta,p(s)} - \eta^H k^H \varepsilon^H_{s,p(s)}
\]

Dividing both sides by \(e^H(p,v,z)\) yields

\[
p_{sS^H} \varepsilon^H_{s,z} \equiv -\eta^H k^H \varepsilon^H_{\eta,p(s)} - \eta^H k^H \varepsilon^H_{s,p(s)}
\]

Let the explicit budget share of sex be \(w^H_s = \frac{p_{sS^H}}{e^H(p,v,z)}\) and the implicit budget share of children be

\[
\omega^H_k = \frac{\eta^H k^H}{e^H(p,v,z)}.
\]

\[
w^H_s \varepsilon^H_{s,z} \equiv -\omega^H_k \varepsilon^H_{\eta,p(s)} - \omega^H_k \varepsilon^H_{s,p(s)} \equiv -\omega^H_k \left[\varepsilon^H_{\eta,p(s)} + \varepsilon^H_{s,p(s)}\right]
\]
This can also be expressed

\[ \varepsilon_{s,z}^H = -\frac{\omega_k^H}{w_z^H} \varepsilon_{\eta,p(s)} - \frac{\omega_k^H}{w_z^H} \varepsilon_{s,p(s)} = -\frac{\omega_k^H}{w_z^H} \left[ \varepsilon_{\eta,p(s)} + \varepsilon_{s,p(s)} \right] \]

Assume that contraception has a relatively little effect on the change in the price ratio, so \( \varepsilon_{\eta,p(s)} \) is close to zero. This result shows that the effect of contraception on Hicksian demand for sex is approximately equal to the effect of a change in the price of sex magnified by the budget share ratio \( \frac{\omega_k^H}{w_z^H} \). Given that implicit cost of children is likely to be large relative to the explicit cost of sex, the contraceptive effect is likely to be much larger than the price effect.

**Demand for Sex and Contraception**

Assume sex is a normal good. The effect of contraceptive effectiveness on the demand for sex is

\[ \frac{\partial s^*}{\partial z} = \frac{\partial s^H}{\partial z} + \beta \eta^H \frac{\partial s^*}{\partial \eta^H} \]

This can also be expressed

\[ \frac{\partial s^*}{\partial z} = -\beta s^H \frac{\partial \eta^H}{\partial p_s} - \beta \eta^H \frac{\partial s^H}{\partial p_s} + \beta \eta^H s^* \frac{\partial s^*}{\partial \eta^H} \]

The three terms are the implicit cost effect, the substitution effect, and the income effect. The substitution and income effects are positive and the implicit cost effect is ambiguous.

**Proof**

The identity for the Hicksian demand is
\[ s^H(p, v, z) \equiv s^*(p, e^H(p, v, z), z) \]

Differentiating with respect to \( z \) shows

\[
\frac{\partial s^H}{\partial z} \equiv \frac{\partial s^*}{\partial m} \frac{\partial e^H}{\partial z} + \frac{\partial s^*}{\partial z}
\]

Substituting the identity \( \frac{\partial e^H}{\partial z} \equiv -\beta \eta^H s^H(p, v, z) \) and rearranging this shows

\[
\frac{\partial s^*}{\partial z} \equiv \frac{\partial s^H}{\partial z} + \beta \eta^H s^H \frac{\partial s^*}{\partial m}
\]

Evaluating this at the point where \( s^H = s^* \) gives the first result. The second result is obtained by substituting in the decomposition of the substitution effect derived earlier.

We have already argued that the \( \frac{\partial s^H}{\partial z} > 0 \). The second term is positive when sex is a normal good, \( \frac{\partial s^*}{\partial m} > 0 \). We share the consensus that inferior goods are members of broader categories of goods that are normal in aggregate. Since sex is not a member of either of the other two categories, it must be a normal good.\(^{24}\)

*Elasticity of Demand for Sex and Contraception*

\[
\frac{\partial s^*}{\partial z} \equiv \frac{\partial s^H}{\partial z} + \beta \eta^H s^H \frac{\partial s^*}{\partial m}
\]

Multiplying both sides by \( \frac{(1-z)}{s^*} \) and the last term by \( \frac{m}{m} \)

\[
\frac{\partial s^*}{\partial z} \frac{(1-z)}{s^*} \equiv \frac{\partial s^H}{\partial z} \frac{(1-z)}{s^*} + \beta \eta^H s^H \frac{\partial s^*}{\partial m} \frac{(1-z)}{s^*} \frac{m}{m}
\]

\(^{24}\) Since sex is not an inferior member of children or other goods, so it is normal. This is unlike, say, fast food, which is an inferior member of restaurant meals, which in aggregate is a normal good.
\[
\frac{\partial s^*}{\partial z} \frac{(1 - z)}{s^*} \equiv \frac{\partial s^H}{\partial z} \frac{(1 - z)}{s^*} + \frac{\eta^H (1 - z)}{m} \frac{\partial s^*}{\partial m} \frac{m}{s^*}
\]

\[
\varepsilon^*_{s,z} \equiv \varepsilon^H_{s,z} + \omega_k \varepsilon^*_{s,m}
\]

The elasticity of sex with respect to contraceptive effectiveness is equal to the elasticity of
Hicksian demand for sex with respect to contraceptive effectiveness plus the income elasticity of
sex times the implicit budget share of children. The implicit budget share of children is the
number of children times the implicit cost as a percent of income.

**Hicksian Demand for Children and Contraception**

The effect of contraceptive effectiveness on the Hicksian demand for children is

\[
\frac{\partial k^H}{\partial z} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_k} - \beta \eta^H \frac{\partial s^H}{\partial p_k}
\]

Both terms are ambiguous.

**Proof**

The proof follows the earlier proof for Hicksian demand for sex.

**Demand for Children and Contraception**

Assume children are normal goods. The effect of contraceptive effectiveness on the demand for
children is

\[
\frac{\partial k^*}{\partial z} \equiv \frac{\partial k^H}{\partial z} + \beta \eta^H s^* \frac{\partial k^*}{\partial m}
\]
The three terms are the implicit cost effect, the substitution effect, and the income effect. The first two terms are ambiguous and the third is positive.

**Proof**

The proof follows the earlier proof for demand for sex.

**Children, Contraception, and the Elasticity of Demand for Sex**

The effect of contraception on demand and Hicksian demand can be expressed as

\[
\frac{\partial k^*}{\partial z} = (1 - z) \beta \frac{\partial s^*}{\partial z} - \beta s^* \\
\frac{\partial k^H}{\partial z} = (1 - z) \beta \frac{\partial s^H}{\partial z} - \beta s^H
\]

Both results signs depend on the elasticity of sex with respect to contraceptive ineffectiveness.

\[
\frac{\partial k^H}{\partial z} > 0 \text{ if and only if } \varepsilon_{s,(1-z)}^H > -1 \\
\frac{\partial k^*}{\partial z} > 0 \text{ if and only if } \varepsilon_{s,(1-z)}^* > -1
\]
If demand for sex is elastic with respect to contraceptive effectiveness, then an increase in contraceptive effectiveness leads to more children.

**Proof**

At the solution to the utility maximization problem,

\[(1 - z)\beta s^* \equiv k^*\]

Differentiating both sides with respect to contraceptive effectiveness, \(z\), gives the first results. The derivative of demand for children with respect to \(z\) is positive when

\[(1 - z)\beta \frac{\partial s^*}{\partial z} > -\beta s^*\]

Dividing both sides by \(\beta s^*\) gives the result

\[\varepsilon^*_{s,(1-z)} \left(\frac{\partial s^* (1 - z)}{\partial z} \frac{1}{s^*}\right) > -1\]

The proof for the Hicksian demand for children follows similarly.

**Hicksian Demand for Other Goods and Contraception**

The effect of contraceptive effectiveness on the Hicksian demand for other goods is

\[
\frac{\partial x^H}{\partial z} \equiv -\beta s^H \frac{\partial \eta^H}{\partial p_x} - \eta^H \frac{\partial s^H}{\partial p_x}
\]

Both terms are ambiguous.
**Proof**

These results follow the earlier proof for Hicksian demand for sex.

**Demand for Other Goods and Contraception**

The effect of contraceptive effectiveness on the Hicksian demand for children is

\[
\frac{\partial x^*}{\partial z} = \frac{\partial x^H}{\partial z} + \beta s^H \frac{\partial x^*}{\partial m}
\]

and

\[
\frac{\partial x^*}{\partial z} = -\beta s^H \frac{\partial \eta^H}{\partial p_x} - \beta \eta^H \frac{\partial s^H}{\partial p_x} + \beta \eta^H s^* \frac{\partial x^*}{\partial m}
\]

The three terms are the implicit cost effect, the substitution effect, and the income effect. The first two terms are ambiguous and the third is positive.

**Proof**

These results follow the earlier proof for demand for sex.